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THE MATHEMATICAL MODELS OF DYNAMICS IN THE INNOVATION PROCESS MANAGEMENT

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Malyarets L. M., Voronin A. V., Lebedeva I. L., Lebedev S. S. The Mathematical Models of Dynamics in the Innovation Process Management

The article considers the main approaches to synergetic management of innovation processes, which determine the sustainable development of the modern economy as a knowledge economy. In this case, economic development is understood as a significantly nonlinear process that occurs in an open system and can have a jump-like nature when transitioning from one stationary state to another or even to a chaotic state. The feasibility of using the synergetic principles to create efficient mechanisms for managing innovation activities is substantiated. The main factors that determine the effectiveness of implementing new technologies and innovative products in the conditions of a knowledge economy are identified. A mathematical model of nonlinear dynamics is proposed that describes the process of new technologies diffusion in self-organizing systems. For this purpose, a logistic curve was used, the determination of which was carried out using an ordinary differential equation with respect to the first-order derivative of a technologically and economically significant indicator characterizing the new technology. Formally, synergistic control is implemented as additional negative feedback in the basic differential equation describing the state of the system. To characterize this control effect, the quadratic function of the significant indicator is used. The analysis of the influence of the parameters of this function on the presence of equilibrium states in the economic system and the stability of these states is carried out. The conditions under which a catastrophic failure of stability may be observed due to the appearance of bifurcations of various nature are determined. Therefore, the presence of nonlinearity in the structure of the controlling influence on the innovation process dynamics radically changes the behavioral properties of the studied system, making it structurally unstable. Based on the conclusions regarding synergistic management, according to the proposed mathematical model, it is possible to prevent negative trends in the evolution of the innovation process to ensure the implementation of the economic strategy of sustainable development.

Keywords: *diffusion of innovations, synergistic management, self-organization, logistic function, feedback.*
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Малярець Л. М., Воронін А. В., Лєбєдєва І. Л., Лебедєв С. С. Математичні моделі динаміки в управлінні інноваційним процесом

У статті розглянуто основні підходи до синергетичного управління інноваційними процесами, що обумовлюють сталий розвиток сучасної економіки як економіки знань. При цьому під економічним розвитком розуміємо істотно нелінійний процес, який відбувається у відкритій системі і може мати стрибкоподібний характер при переході з одного стаціонарного стану в інший або навіть у хаотичний стан. Обґрунтовано доцільність використання принципів синергетики для формування ефективних механізмів управління інноваційною діяльністю. Визначено основні фактори, що впливають на результативність імплементації новітніх технологій та інноваційних продуктів в умовах економіки знань. Запропоновано математичну модель нелінійної динаміки, що описує процес еволюції новітніх технологій у системах із самоорганізацією. Для цього було застосовано логістичну криву, визначення якої здійснювалось за допомогою звичайного диференціального рівняння відносно похідної першого порядку від технологічно та економічно значущого показника, що характеризує новітню технологію. Формально синергетичне управління реалізовано як додатковий негативний зворотний зв'язок у базовому диференціальному рівнянні, що описує стан системи. Для характеристики цього

керуючого впливу застосовується квадратична функція значущого показника. Проведено аналіз впливу параметрів цієї функції на наявність в економічній системі рівноважних станів та стійкість цих станів. Визначено умови, при яких може спостерігатися катастрофічний зрив стійкості, що пов'язано з появою біфуркацій різної природи. Отже, наявність нелінійності в структурі керуючого впливу на динаміку інноваційного процесу докорінно змінює поведінкові властивості досліджуваної системи, роблячи її структурно нестійкою. Спираючись на висновки щодо синергетичного управлінні, згідно із запропонованою математичною моделлю можна запобігти негативним тенденціям еволюції інноваційного процесу для забезпечення реалізації економічної стратегії сталого розвитку.
Ключові слова: *дифузія інновацій, синергетичне управління, самоорганізація, логістична функція, зворотний зв'язок.*
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The modern state of the economy is defined as a knowledge economy. In this context, humanity faces the challenge of finding innovative paths for development, since the creation of cutting-edge technologies and their effective implementation in all spheres of life enables sustainable development not only of the economy but also of society as a whole. Throughout the 20th century, the conception of sustainable development has undergone significant changes, shifting from its initial focus on the environmental dimension of economic growth to its contemporary understanding, which is presented in the form of the 17 United Nations Sustainable Development Goals [1], adopted in September 2015. In light of these global goals, sustainable development should be understood as a structural synthesis of the imperatives of economic, environmental, and social stable evolutions, whose course is coordinated at the global, national, and regional levels. Thus, the conception of sustainable development, which is a new paradigm of the 21st century, currently has a systemic synergistic nature, as it involves coordinated actions within an open system of a certain level.

In this study, economic development will be understood as a substantially nonlinear process that can occur in the form of abrupt transitions from one stationary state to another. Traditional linear models often do not allow for accounting for rapid changes in market trends, irregular and unpredictable fluctuations observed in real economic systems [2; 3]. Advanced mathematical models that consider the nonlinear dependence of the system's state on initial conditions provide a more accurate understanding of the complex dynamics of the evolution of technological and socioeconomic systems [4–6, etc.]. Thus, for forecasting innovation-driven development to be effective, it should be based on Kondratiev's theory of prediction [7; 8] and Schumpeter's theory of innovation [9], which are currently elaborated in the works of numerous contemporary scholars [10–13, etc.].

A fundamental prerequisite for substantiating qualitative forecasting of processes of any kind is the concept that predicting evolutionary paths relies on calculating the interaction of statistical indicators that determine the state of equilibrium and cyclic dynamics, where coherence of cycles of different durations occurs, as well as sociogenesis, i. e., the formation of social consciousness. In response to the demands of

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the time, a new integrative science is now emerging – synergetics – which studies self-organization processes and encompasses nearly all fields of modern knowledge about nature, technology, and economics. This generalizing interdisciplinary science is based on nonlinear dynamics and the thermodynamics of irreversible processes.

There are two main approaches to understanding synergy as a natural phenomenon. The first approach was proposed by H. Haken [14; 15], and the second by I. Ansoff [16]. The emergence of synergetics as an independent scientific field dates back to 1969. It was then that the German physicist Hermann Haken began using the term «synergetics» when teaching a course on the theory of laser radiation. The essence of this scientific field is that in open systems, which exchange energy, matter, and information with the environment, processes of spontaneous self-organization occur – that is, processes in which stable, ordered structures with new properties emerge from chaos. Furthermore, it is important to emphasize that nonlinearity, in a broad sense, implies that the result of the interaction of system elements exceeds the sum of the results of these elements acting individually. In other words, instead of additivity, multiplicativity occurs. This should be taken into account when constructing mathematical models of such systems [17–19, etc.]. Although the foundations of synergetics as a scientific discipline were developed within theoretical physics, these ideas are effectively applied in engineering, biology (swarm effect and butterfly effect) [20], sociology (team effect) [21; 22], economics [23–25], management theory [26], and so on. In general, the synergetic effect manifests as an increase in the efficiency of resource use, not only material but also immaterial [27; 28].

Effective application of synergetic ideas in the management of economic entities requires a fundamentally new perspective on the nature of management and self-governance processes. This new perspective implies a shift from the notion of the unpredictability of an economic system's behavior as a dissipative structure to its directed movement along desired trajectories – attractors. An attractor refers to a set of states of a dynamical system – points of attraction toward which the system moves over time along phase trajectories, that is, a subset of states in the phase space of a dynamical system that the system tends toward regardless of initial conditions [29–31 et al.]. In economics, an attractor is an innovative solution that rapidly changes the market conjuncture and promotes the allocation of resources to a new direction of development. Other characteristics of the dynamical system also adjust to these changes, so the system itself synthesizes this targeted self-organization. According to this approach, an attractor defines the essence of a process, which involves self-governance according to an established goal. Within the theory of dynamical systems, this means guiding the corresponding system to a specific final state regardless of its initial state. Accordingly, the perspective on the management problem changes, there is a gradual transition to modern ideas of synergetics, and a synergetic management theory is developed as an embodiment of the principles of self-organization in addressing management issues [32–34]. The main features of the synergetic management theory are, first, a fundamental change in the goals of the behavior of synthesized economic systems, second, the direct consideration of the natural properties of nonlinear objects in synthesis procedures, and third, the formation of a new mechanism for generating feedback.

Unlike the classical theory of optimal control, in the synergetic approach to synthesizing the goals of economic development, the operation of a closed nonlinear system involves not only fulfilling certain requirements regarding the nature of the transitional process but, primarily, ensuring the desired asymptotic convergence of the original system to the attractor. This is due to the circumstances that the behavior of any nonlinear dissipative system can be divided into a stage of transient motion, when its trajectories move toward an attractor, and a stage of asymptotic motion on the desired attractor. This approach allows for fundamentally solving the problem of analytically synthesizing objective laws for controlling nonlinear objects.

In particular, all the above-mentioned points sufficiently apply to many economic processes with quadratic nonlinearities, which are defined by so-called logistic curves. In this case, the logistic equation can be considered a universal tool for describing real economic processes and phenomena, with which it is possible to analyze the patterns of evolution of economic systems and ensure efficient forecasting.

The aim of this article is the construction of a mathematical model of a nonlinear regulator that determines the diffusion of the innovation process. The use of such a regulator allows ensuring the efficiency of synergetic management, the purpose of which is to bring the system to a predetermined level (equilibrium value) of a characteristic indicator that reflects the diffusion of an innovative product, and also creates conditions to prevent the emergence of undesirable bifurcations and catastrophes.

The evolution process of the newest technologies (innovations) can, in its simplest form, be described using a logistic curve, which is defined by an ordinary differential equation of the following type:

$$\frac{dx}{dt} = \gamma(x-a)(b-x),\ \gamma > 0, \qquad \textbf{(1)}$$

where t is time, i. e., a variable that reflects the moments when economic agents incur expenditures on the development of new equipment or technology;

$x = x(t)$ is a technologically and economically significant indicator characterizing this new technology (innovation);

γ is a scale parameter, the reciprocal of which represents the characteristic time of the transitional process in the original model;

a and b are positive constants that bound the value (from below and above, respectively) of the characteristic significant indicator $x(t)$ of the functioning of a given technology. Here, a is the lower bound of $x(t)$, representing the initial (starting) extremely low capabilities of implementing the latest technology, and b is the technological limit or potential, reflecting the possibilities of $x(t)$ maximum growth [35].

As expenditures increase, regardless of how they are measured, for the implementation and further improvement of the latest technology (innovation), its characteristic significant indicator can only increase. Therefore, the function $x = x(t)$ is smooth and increases over its entire domain. The fact that the growth rate of the quantity $x(t)$, according to the internal logic of equation (1), is directly proportional to the distance of this quantity from its initial capabilities indicates that the function $x = x(t)$ grows faster the greater this distance. On the other hand, the growth rate of the function $x = x(t)$ is proportional to the magnitude of $(b - x)$, which implies a slowdown in the growth rate of the function $x = x(t)$ as the exponent value of $x(t)$ approaches its technological limit. It can be stated that the differential equation (1) is a mathematical formalization of the law governing the mutual transition of quantitative and qualitative changes during the implementation of cumulative processes in the economy.

The equation (1), supplemented by the initial condition $x(t = 0) = x_0$, has an explicit solution in time:

$$x(t) = a + \frac{(b-a)f(t)}{f(t)+K}, \qquad \textbf{(2)}$$

where $f(t) = \exp\left(\left(b-a\right)\gamma t\right)$;

K is an arbitrary constant that depends on the initial condition $x(t = 0) = x_0$.

Furthermore, the differential equation (1) has two singular solutions: $x_1^* = a$ and $x_2^* = b$. Concerning the particular solutions, which represent equilibrium positions, let us note that $x_1^* = a$ is an unstable equilibrium, whereas $x_2^* = b$ on the contrary, is a stable equilibrium. The economic significance of these facts is contained in the previously presented description of the parameters a and b of the original differential equation (1). Thus, formula (2), along with the corresponding commentary, provides a comprehensive description of the phenomenon of self-organization in the process of innovation diffusion.

Let us consider a situation where it is necessary to influence the dynamics of an evolutionary process. The need for this step can be justified by various reasons. We may not be fully satisfied with the values of the equilibrium states of the process under study and may want to change them in a desired direction through some external influence. In this case, the goal of managing the system is to reach an attractor corresponding to a new equilibrium position. A substantive reason for introducing a controlling influence on the process of innovative self-organization may be either the entrepreneurial initiative of innovators or the need to improve the efficiency of measures for State regulation of innovation activities. In a synergetic economy, positive and negative feedback loops play a key role in shaping development dynamics. Positive feedbacks typically lead to accelerated changes and explosive growth, whereas negative feedbacks serve a stabilizing function, preventing the development of bifurcations of various kinds and balancing the state of the system. Both positive and negative feedbacks can be considered important factors that enhance the properties of an open nonlinear system [36–38].

Formally, we will implement control as an additional negative feedback in the basic system equation (1). Then the differential equation (1) with $\gamma = 1$ takes the form:

$$\frac{dx}{dt} = (x-a)(b-x) - u, \qquad \textbf{(3)}$$

where $u = \varphi(x)$ is the nonlinear control influence. We will assume that the function $\varphi(x)$ is quadratic with respect to the variable x and can be explicitly represented as:

$$\varphi(x) = \varphi_0 + \varphi_1 x + \varphi_2 x^2, \qquad \textbf{(4)}$$

where φ_0 and φ_1 are only positive numbers, and the sign of φ_2 can be arbitrary. We deliberately do not fix the sign of φ_2, to describe deviations from linear control in both directions.

After substituting the function $u = \varphi(x)$ in explicit form (4) into differential equation (3), we obtain the following differential equation:

$$\frac{dx}{dt} = -(\varphi_0 + ab) + (a + b - \varphi_1)x - (\varphi_2 + 1)x^2. \qquad \textbf{(5)}$$

Clearly, the structure of the singular points (equilibrium positions) of the differential equation (5) is determined by the solutions of the quadratic equation:

$$(\varphi_2 + 1)x^2 + (\varphi_1 - a - b)x + \varphi_0 + ab = 0, \qquad \textbf{(6)}$$

which has the discriminant of:

$$D = (\varphi_1 - a - b)^2 - 4(\varphi_2 + 1)(\varphi_0 + ab). \quad (7)$$

From expressions (6) and (7), it follows that there are two positive equilibrium positions if the condition $D > 0$ for equation (5) is satisfied. If $D < 0$ the equilibrium position is absent. Furthermore, a double equilibrium position may appear under condition of $D = 0$.

Suppose there is an inequality of $\varphi_2 + 1 > 0$, i. e. $\varphi_2 > -1$. Then, similar to differential equation (1), it can be shown that in (5) the larger equilibrium position is stable, while the smaller one is unstable.

When the distance between the two singular points of equation (5) is a small value, the system under study exhibits complex behavior. In fact, structural instability occurs as a consequence of a saddle-node bifurcation. For a specific analysis of the topological properties of this bifurcation, we transform equation (5) into a form corresponding to the normal one. To do this, we introduce a new variable that depends linearly on the original variable:

$$x(t) = \frac{2y(t) + a + b - \varphi_1}{2(\varphi_2 + 1)}. \quad (8)$$

As a result of the transformations, we obtain:

$$\frac{dy}{dt} = \alpha - y^2, \quad (9)$$

where $\alpha = \dfrac{\varphi_0 + ab}{\varphi_2 + 1} - \dfrac{(\varphi_1 - a - b)^2}{4(\varphi_2 + 1)^2}$.

The differential equation (9) is the normal form of a saddle-node bifurcation [39; 40, etc.] provided that the magnitude of α is small. Since $\alpha = \dfrac{-D}{4(\varphi_2 + 1)^2}$, the equation (9) describes the behavior of model (5) in a small neighborhood of a double equilibrium position.

Thus, at $\alpha > 0$ the system (9) has two equilibrium positions – stable and unstable. At $\alpha = 0$ these two equilibrium positions merge, they form a single «semi-stable» equilibrium, and at $\alpha < 0$ no equilibrium positions exist in the system.

Let us examine more closely the behavior of the stable equilibrium. As the coefficient of the linear controller φ_1 approaches its critical value, at $\alpha = 0$ an «equilibrium breakdown» occurs with a catastrophic loss of stability. This corresponds to a type of catastrophe known as a «fold-down» [41; 42 etc.].

If $\varphi_2 + 1 < 0$, that is, $\varphi_2 < -1$, it is easy to show that a transcritical bifurcation occurs, which has two branches with different tangents, the so-called «stability exchange» bifurcation. If we introduce one more assumption that with a negative coefficient φ_2 may be negative and the following conditions are satisfied:

$$\varphi_0 = -ab, \; \varphi_1 = a + b, \quad (10)$$

then the differential equation (5) takes the form:

$$\frac{dx}{dt} = -(\varphi_2 + 1)x^2. \quad (11)$$

The differential equation (11) is a typical example of a system with strong positive feedback and can describe a regime with exacerbation. The solution of equation (11) is written in the form:

$$x(t) = \frac{x_0}{1 + x_0(\varphi_2 + 1)t}. \quad (12)$$

It is obvious that the solution (12) exists only up to the time instant of $t^* = -\dfrac{1}{x_0(\varphi_2 + 1)}$. Moreover, the moment of escalation itself depends on the initial value of x_0. The larger the x_0, the shorter the duration of existence of the solution. This means that a certain real process can, for some time, be described by equations whose solutions grow in an escalation regime. In that case, trying to predict the course of a linear or more complex interpolation is doomed to fail. At this, the approach of «planning based on achievements» and the logic of «tomorrow will be almost the same as today» become inapplicable.

CONCLUSIONS

Based on the analysis conducted, one can reach the sobering conclusion that the presence of nonlinear control influences on the dynamics of the innovation process fundamentally changes the behavioral properties of the system under study, making it structurally unstable. This situation triggers the occurrence of undesirable bifurcations, accompanied by a catastrophic loss of stability. Therefore, the most important aim in establishing management rules is to prevent negative trends in the evolution of the innovation process while implementing a sustainable development economic strategy. ■

BIBLIOGRAPHY

1. 17 Sustainable Development Goals. *Global Compact. Network Ukraine*. URL: https://globalcompact.org.ua/en/17-sustainable-development-goals/
2. Shah Z., Bonyah E., Alzahrani E. et al. Chaotic phenomena and oscillations in dynamical behaviour of financial system via fractional calculus. *Complexity*. 2022. P. 1–14. DOI: https://doi.org/10.1155/2022/8113760
3. Zhou S. S., Jahanshahi H., Din Q. et al. Discrete-time macroeconomic system: Bifurcation analysis and synchronization using fuzzy-based activation feedback control. *Chaos, Solitons & Fractals*. 2021. Vol. 142. Art. 110378. DOI: https://doi.org/10.1016/j.chaos.2020.110378
4. Chen S.-B., Jahanshahi H., Abba O. A. et al. The effect of market confidence on a financial system from

the perspective of fractional calculus: Numerical investigation and circuit realization. *Chaos, Solitons & Fractals*. 2020. Vol. 140. Art. 110223.
DOI: https://doi.org/10.1016/j.chaos.2020.110223
5. Gardini L., Lamantia F., Radi D. et al. Nonlinear dynamics in economic modelling. *Decisions in Economics and Finance*. 2021. Vol. 44. P. 485–487.
DOI: https://doi.org/10.1007/s10203-021-00353-8
6. Voronin A., Lebedeva I., Lebedev S. A nonlinear mathematical model of dynamics of production and economic objects. *Development Management*. 2022. Vol. 21. No. 2. P. 8–15.
DOI: https://doi.org/10.57111/devt.20(2).2022.8-15
7. Kondratieff N. D. The Long Waves in Economic Life. *The Review of Economic Statistics*. 1935. Vol. 17. No. 6. P. 105–115.
DOI: https://doi.org/10.2307/1928486
8. Kondratieff N. D. The Long Wave Cycle. New York: Richardson & Snyder, 1984. 138 p.
9. Shumpeter Y. A. Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process. New York : McGraw-Hill Book Company, 1939.
10. Zhang W.-B. Synergetic Economics: Time and Change in Nonlinear Economics. New York : Springer-Verlag, 1991.
11. Mikhailov A. S., Loskutov A. Yu. Foundations of Synergetics II: Complex Patterns (Springer Series in Synergetics). New York : Springer-Verlag, 1991.
12. Grinin L., Korotayev A., Tausch A. Economic Cycles, Crises, and the Global Periphery. New York : Springer Cham., 2016.
DOI: https://doi.org/10.1007/978-3-319-41262-7
13. Якімцов В. В. Синергетичні дослідження в еконок міці: проблеми та перспективи. Львів : ВД «Панорама», 2018. 448 с.
14. Haken H. Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems and Devices. NewYork : Springer-Verlag, 1993.
15. Yakimtsov V. V. History and Development of Haken's Synergetics. *Scientific Bulletin of UNFU*. 2018. Vol. 28. No. 9. P. 119–125.
DOI: https://doi.org/10.15421/40280923
16. Ansoff H. I. Strategic Management. London : Macmillan academic and professional LTD, 1979.
DOI: https://doi.org/10.1007/978-1-349-02971-6
17. Santillan M., Zeron E. S. Analytical Study of the Multiplicity of Regulatory Mechanisms in the Tryptophan Operon. *Bulletin of Mathematical Biology*. 2006. Vol. 68. P. 343–359.
DOI: https://doi.org/10.1007/s11538-005-9025-0
18. Malinsky G. G. Synergetics – From Past to Future. *Modeling and Analysis of Information Systems*. 2012. Vol. 19. Iss. 3. P. 5–31.
DOI: https://doi.org/10.18255/1818-1015-2012-3-5-31
19. Yerznkyan B., Gataullin S., Gataullin T. Mathematical Aspects of Synergy. *Montenegrin Journal of Economics*. 2022. Vol. 18. No. 3. P. 197–207.
DOI: https://doi.org/10.14254/1800-5845/2022.18-3.16
20. Otto K.-S., Nolting U., Bassler C. Evolutionsmanagement – Von der Natur lernen: Unternehmen entwickeln und langfristig steuern. Hanser Wirtschaftsbücher, München, 2007.
21. Voznyuk A., Kubitskyi S., Balanovska T. et al. Synergetic simulation of managing processes in educational sphere in the contest of temporary self-ruled managerial target teams application. *Financial and Credit Activity Problems of Theory and Practice*. 2022. No. 3. P. 317–327.
DOI: https://doi.org/10.55643/fcaptp.3.44.2022.3749
22. Voznyuk A., Zdanevych L. Application of System and Synergetic Paradigm of Management of Social-Economic, Educational Processes in Ukraine. *Pedagogical Discourse*. 2019. Iss. 26. P. 19–26.
DOI: https://doi.org/10.31475/ped.dys.2019.26.03
23. Nijkamp P. Poot J. Lessons from Nonlinear Dynamics Economics. In: *Nonlinear Evolution of Spatial Economic Systems* / Nijkamp P. and Reggiani A. (Eds.). Springer, Berlin, Heidelberg, New York, 1993.
DOI: http://dx.doi.org/10.1007/978-3-642-78463-7_2
24. Коломієць С. В. Категорії синергетики в економічо них дослідженнях: нелінійність соціально-економічних систем. *Вчені Записки ТНУ імені В. І. Вернадського. Серія «Економіка і Управління»*. 2020. Т. 31. № 3. С. 191–197.
DOI: https://doi.org/10.32838/2523-4803/70-3-66
25. Sieja M., Wach K. The Use of Evolutionary Algorithms for Optimization in the Modern Entrepreneurial Economy: Interdisciplinary Perspective. *Entrepreneurial Business and Economics Review*. 2019. Vol. 7. No. 4. P. 117–130.
DOI: https://doi.org/10.15678/EBER.2019.070407
26. Zeng S. X., Shi J. J., Lou G. X. A synergetic model for implementing an integrated management system: an empirical study in China. *Journal of Cleaner Production*. 2007. Vol. 15. Iss. 18. P. 1760–1767.
DOI: https://doi.org/10.1016/j.jclepro.2006.03.007
27. Itami H., Numagami T. Dynamic interaction between strategy and technology. *Strategic Management Journal*. 1992. Vol. 13. Iss. 52. P. 119–135.
DOI: https://doi.org/10.1002/smj.4250130909
28. Liening A. Synergetics – Fundamental Attributes of the Theory of Self-Organization and Its Meaning for Economics. *Modern Economy*. 2014. Vol. 5. No. 8. P. 841–847.
DOI: https://doi.org/10.4236/me.2014.58077
29. Ruelle D., Taken F. On the nature of turbulence. *Communications in Mathematical Physics*. 1971. Vol. 20. P. 167–192.
DOI: https://doi.org/10.1007/BF01646553
30. Hammouch Z., Yavuz M., Özdemir N. Numerical solutions and synchronization of a variable-order fractional chaotic system. *Mathematical Modeling and Numerical Simulation with Applications*. 2021. Vol. 1. Iss. 1. P. 11–23.
DOI: https://doi.org/10.53391/mmnsa.2021.01.002

31. Verne J.-F. Relevance of chaos and strange attractors in the Samuelson-Hicks oscillator. *Economic Thought*. 2021. Vol. 10. Iss. 1. P. 32–45. URL: https://hdl.handle.net/10419/315835
32. Djennoune S., Bettayeb M. Optimal synergetic control for fractional-order systems. *Automatica*. 2013. Vol. 49. Iss. 7. P. 2243–2249.
DOI: https://doi.org/10.1016/j.automatica.2013.04.007
33. Заїка В. І., Кишенько В. Д. Синергетичний синтез ієрархічної системи керування технологічним комплексом цукрового заводу. *Восточно-Европейский журнал передовых технологий*. 2013. Вип. 4(2). С. 46–51. URL: https://journals.uran.ua/eejet/article/view/16658/14150
34. Sidikov I. H., Usmanov K. I., Yakubova N. S. Synergetic control of nonlinear dynamic objects. *Chemical Technology. Control and Management*. 2020. No. 2. P. 49–55.
DOI: https://doi.org/10.34920/2020.2.49-55
35. Haken H. Chapter II. Mathematical Methods of Synergetics for Applications to Self-Organizing Systems. *North-Holland Mathematics Studies*. 1982. Vol. 58. P. 9–14.
DOI: https://doi.org/10.1016/S0304-0208(08)71226-X
36. Cinquin O., Demongeot J. Roles of positive and negative feedback in biological systems. *Comptes Rendus Biologies*. 2002. Vol. 325. Iss. 11. P. 1085–1095.
DOI: https://doi.org/10.1016/S1631-0691(02)01533-0
37. Zeron E. S. Positive and Negative Feedback in Engineering and Biology. *Mathematical Modelling of Natural Phenomena*. 2008. Vol. 3. No. 2. P. 67–84.
DOI: https://doi.org/10.1051/mmnp:2008055
38. Lucky A. C., Iyai D., Stanley C. T., Humphrey A. E. Stability and feedback control of nonlinear systems. *International Journal of Control Science and Engineering*. 2020. Vol. 10. Iss. 1. P. 11–15.
DOI: 10.5923/j.control.20201001.02
39. Hsiao Y.-C., Tung P.-C. Mechanism of producing a saddle-node bifurcation with the coalescence of two unstable periodic orbits. *Chaos, Solitons & Fractals*. 2002. Vol. 13. Iss. 7. P. 1429–1438.
DOI: https://doi.org/10.1016/S0960-0779(01)00147-3
40. Glendinning P. A., Simpson D. J. W. Normal forms for saddle-node bifurcations: Takens' coefficient and applications in climate models. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*. 2022. Vol. 478.
DOI: https://doi.org/10.1098/rspa.2022.0548
41. Kuznetsov Yu. A., Meijer H. G. E., van Veen L. The fold-flip bifurcation. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*. 2004. Vol. 14. No. 7. P. 2253–2282.
DOI: https://doi.org/10.1142/S0218127404010576
42. Habib G., Horváth A. Fold bifurcation identification through scientific machine learning. *Physica D: Nonlinear Phenomena*. 2025. Vol. 472.
DOI: https://doi.org/10.1016/j.physd.2024.134490

REFERENCES

17 Sustainable Development Goals. (n. d.). Global Compact. Network Ukraine. https://globalcompact.org.ua/en/17-sustainable-development-goals/

Ansoff, H. I. (1979). *Strategic Management*. Macmillan academic and professional LTD.
https://doi.org/10.1007/978-1-349-02971-6

Chen, S.-B., Jahanshahi, H., Abba, O. A., Bekiros, S., & Ozdemir, O. (2020). The effect of market confidence on a financial system from the perspective of fractional calculus: Numerical investigation and circuit realization. *Chaos, Solitons & Fractals, 140*, Art. 110223.
https://doi.org/10.1016/j.chaos.2020.110223

Cinquin, O., & Demongeot, J. (2002). Roles of positive and negative feedback in biological systems. *Comptes Rendus Biologies, 325*(11), 1085–1095.
https://doi.org/10.1016/S1631-0691(02)01533-0

Djennoune, S., & Bettayeb, M. (2013). Optimal synergetic control for fractional-order systems. *Automatica, 49*(7), 2243–2249.
https://doi.org/10.1016/j.automatica.2013.04.007

Gardini, L., Lamantia, F., Radi, D., & Sushko, I. (2021). Nonlinear dynamics in economic modelling. *Decisions in Economics and Finance, 44*, 485–487.
https://doi.org/10.1007/s10203-021-00353-8

Glendinning, P. A., & Simpson, D. J. W. (2022). Normal forms for saddle-node bifurcations: Takens' coefficient and applications in climate models. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 478*.
https://doi.org/10.1098/rspa.2022.0548

Grinin, L., Korotayev, A., & Tausch, A. (2016). *Economic Cycles, Crises, and the Global Periphery*. Springer Cham.
https://doi.org/10.1007/978-3-319-41262-7

Habib, G., & Horváth, A. (2025). Fold bifurcation identification through scientific machine learning. *Physica D: Nonlinear Phenomena, 472*.
https://doi.org/10.1016/j.physd.2024.134490

Haken, H. (1982). Chapter II. Mathematical Methods of Synergetics for Applications to Self-Organizing Systems. *North-Holland Mathematics Studies, 58*, 9–14.
https://doi.org/10.1016/S0304-0208(08)71226-X

Haken, H. (1993). *Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems and Devices*. Springer-Verlag.

Hammouch, Z., Yavuz, M., & Özdemir, N. (2021). Numerical solutions and synchronization of a variable-order fractional chaotic system. *Mathematical Modeling and Numerical Simulation with Applications, 1*(1), 11–23.
https://doi.org/10.53391/mmnsa.2021.01.002

Hsiao, Y.-C., & Tung, P.-C. (2002). Mechanism of producing a saddle-node bifurcation with the coalescence of two unstable periodic orbits. *Chaos, Solitons & Fractals, 13*(7), 1429–1438.
https://doi.org/10.1016/S0960-0779(01)00147-3

Itami, N., & Numagami, T. (1992). Dynamic interaction between strategy and technology. *Strategic Management Journal, 13*(52), 119–135.
https://doi.org/10.1002/smj.4250130909

Kolomiiets, S. V. (2020). Katehorii synerhetyky v ekonomichnykh doslidzhenniakh: neliniinist sotsialno-ekonomichnykh system [Categories of synergetics in economic researches: nonlinearity of socio-eco-

nomic systems]. *Vcheni Zapysky TNU imeni V. I. Vernadskoho. Seriia «Ekonomika i Upravlinnia», 31*(3), 191–197.
https://doi.org/10.32838/2523-4803/70-3-66

Kondratieff, N. D. (1935). The Long Waves in Economic Life. The Review of Economic Statistics, 17(6), 105–115.
https://doi.org/10.2307/1928486

Kondratieff, N. D. (1984). *The Long Wave Cycle*. Richardson & Snyder.

Kuznetsov, Yu. A., Meijer, H. G. E., & van Veen, L. (2004). The fold-flip bifurcation. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering, 14*(7), 2253–2282.
https://doi.org/10.1142/S0218127404010576

Liening, A. (2014). Synergetics – Fundamental Attributes of the Theory of Self-Organization and Its Meaning for Economics. *Modern Economy,* 5(8), 841–847.
https://doi.org/10.4236/me.2014.58077

Lucky, A. C., Iyai, D., Stanley, C. T., & Humphrey, A. E. (2020). Stability and feedback control of nonlinear systems. *International Journal of Control Science and Engineering, 10*(1), 11–15.
https://doi.org/10.5923/j.control.20201001.02

Malinsky, G. G. (2012). Synergetics – From Past to Future. *Modeling and Analysis of Information Systems*, *19*(3), 5–31.
https://doi.org/10.18255/1818-1015-2012-3-5-31

Mikhailov, A. S., & Loskutov, A. Yu. (1991). *Foundations of Synergetics II: Complex Patterns (Springer Series in Synergetics)*. Springer-Verlag.

Nijkamp, P., & Poot, J. (1993). Lessons from Nonlinear Dynamics Economics. In P. Nijkamp & A. Reggiani (Eds.), *Nonlinear Evolution of Spatial Economic Systems* (p. 11–26). Springer.
https://doi.org/10.1007/978-3-642-78463-7_2

Otto, K.-S., Nolting, U., & Bassler, C. (2007). *Evolutionsmanagement – Von der Natur lernen: Unternehmen entwickeln und langfristig steuern*. Hanser Wirtschaftsbücher.

Ruelle, D., & Taken, F. (1971). On the nature of turbulence. *Communications in Mathematical Physics, 20*, 167–192.
https://doi.org/10.1007/BF01646553

Santillan, M., & Zeron, E. S. (2006). Analytical Study of the Multiplicity of Regulatory Mechanisms in the Tryptophan Operon. *Bulletin of Mathematical Biology, 68*, 343–359.
https://doi.org/10.1007/s11538-005-9025-0

Shah, Z., Bonyah, E., Alzahrani, E., Kumar, S., & Abuasbeh, A. (2022). Chaotic phenomena and oscillations in dynamical behaviour of financial system via fractional calculus. *Complexity*, 2022, 1–14.
https://doi.org/10.1155/2022/8113760

Shumpeter, Y. A. (1939). *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process.* McGraw-Hill Book Company.

Sidikov, I. H., Usmanov, K. I., & Yakubova, N. S. (2020). Synergetic control of nonlinear dynamic objects. Chemical Technology. *Control and Management*, *(2)*, 49–55.
https://doi.org/10.34920/2020.2.49-55

Sieja, M., & Wach, K. (2019). The Use of Evolutionary Algorithms for Optimization in the Modern Entrepreneurial Economy: Interdisciplinary Perspective. *Entrepreneurial Business and Economics Review, 7*(4), 117–130.
https://doi.org/10.15678/EBER.2019.070407

Verne, J.-F. (2021). Relevance of chaos and strange attractors in the Samuelson-Hicks oscillator. *Economic Thought, 10*(1), 32–45.
https://hdl.handle.net/10419/315835

Voronin, A., Lebedeva, I., & Lebedev, S. (2022). A nonlinear mathematical model of dynamics of production and economic objects. *Development Management, 21*(2), 8–15.
https://doi.org/10.57111/devt.20(2).2022.8-15

Voznyuk, A., Kubitskyi, S., Balanovska, T., Prokopenko, O., & Voloshchuk, V. (2022). Synergetic simulation of managing processes in educational sphere in the contest of temporary self-ruled managerial target teams application. *Financial and Credit Activity Problems of Theory and Practice, 3*(44), 317–327.
https://doi.org/10.55643/fcaptp.3.44.2022.3749

Voznyuk, A., & Zdanevych, L. (2019). Application of System and Synergetic Paradigm of Management of Social-Economic, Educational Processes in Ukraine. *Pedagogical Discourse, 26*, 19–26.
https://doi.org/10.31475/ped.dys.2019.26.03

Yakimtsov, V. V. (2018). History and Development of Haken's Synergetics. *Scientific Bulletin of UNFU, 28*(9), 119–125.
https://doi.org/10.15421/40280923

Yakimtsov, V. V. (2018). *Synerhetychni doslidzhennia v ekonomitsi: problemy ta perspektyvy* [Synergetic researches in economy: Problems and prospects]. VD «Panorama».

Yerznkyan, V., Gataullin, S., & Gataullin, T. (2022). Mathematical Aspects of Synergy. *Montenegrin Journal of Economics, 18*(3), 197–207.
https://doi.org/10.14254/1800-5845/2022.18-3.16

Zaika, V. I., & Kyshenko, V. D. (2013). Synerhetychnyi syntez iierarkhichnoi systemy keruvannia tekhnolohichnym kompleksom tsukrovoho zavodu [Synergetic synthesis of hierarchical control system of technological complex of sugar factory]. *Vostochno-Evropeiskyi zhurnal peredovykh tekhnolohyi, 4*(2), 46–51. https://journals.uran.ua/eejet/article/view/16658/14150

Zeng, S. X., Shi, J. J., & Lou, G. X. (2007). A synergetic model for implementing an integrated management system: an empirical study in China. *Journal of Cleaner Production, 15*(18), 1760–1767.
https://doi.org/10.1016/j.jclepro.2006.03.007

Zeron, E. S. (2008). Positive and Negative Feedback in Engineering and Biology. *Mathematical Modelling of Natural Phenomena, 3*(2), 67–84.
https://doi.org/10.1051/mmnp:2008055

Zhang, W.-B. (1991). *Synergetic Economics: Time and Change in Nonlinear Economics*. Springer-Verlag.

Zhou, S. S., Jahanshahi, H., Din, Q., Abidini, M. A., & Baleanu, D. (2021). Discrete-time macroeconomic system: Bifurcation analysis and synchronization using fuzzy-based activation feedback control. *Chaos, Solitons & Fractals, 142*, Art. 110378.
https://doi.org/10.1016/j.chaos.2020.110378